Final-December 11, 1998
EE524-Dr. Dickerson
Answer 4 out of 5 problems

Name:

Code Name for posting grade on web:

Social Security Number

Problem Number	Points	Score
1	25	
2	25	
3	25	
4	25	
5	25	

Name:

1. Linear Phase

A type 3 linear phase filter has an antisymmetric impulse response and an odd length.The condition on the impulse response coefficients is:

$$h[n] = -h[N-n] \qquad 0 \le n \le N$$

a) For N = 6 show that a filter that meets this symmetry condition must be linear phase. Calculate the group delay for this filter. (10 points)

b) Prove that the zeros for this anti-symmetric linear phase filter must be in complex conjugate reciprocal pairs when the filter coefficients are real. (10 points)

c) Can an all pass FIR filter be linear phase - why or why not? (5 points)

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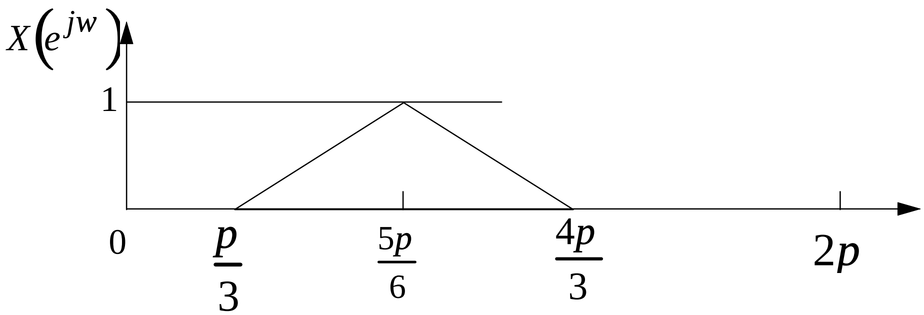
2. Multirate Signal Processing

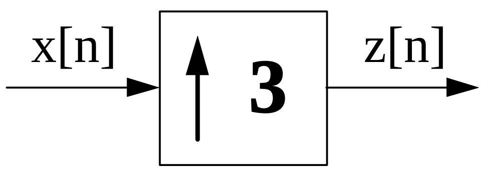
Consider a sequence $x[n]$ with the spectrum shown below.



a) Let $y[n] = x[3n]$. What is the spectrum of $y[n]$ in terms of $X(e^{jw})$? Sketch the spectrum.(10 points)

b) Show how to recover $x[n]$ from $y[n]$ using filters and multirate building blocks.(10 points)

c) Sketch the spectrum of z[n] in terms of $X(e^{jw})$. (5 points)



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3. DFTs

A signal consisting of a single sinusoid of frequency 2.0 Khz is sampled at 8 kHz.

a) 32 samples of the signal are taken and the 32 point DFT is calculated. Give a definition of resolution in the DFT. What is the resolution in the 32 point DFT. Sketch the magnitude of the DFT. (10%)

b) Now zero pad the sequence to length 128 points and take the 128 point DFT. What is the resolution now? Sketch the DFT magnitude. (5%)

c) Take the result of step (b) (the zero-padded DFT $X_p(k)$) and form the sequence:

$$\overline{X}(k) = X_p(2k) \qquad k = 0,1,2,\ldots,63$$

Take the 64 point inverse DFT. Sketch the result. (10%)

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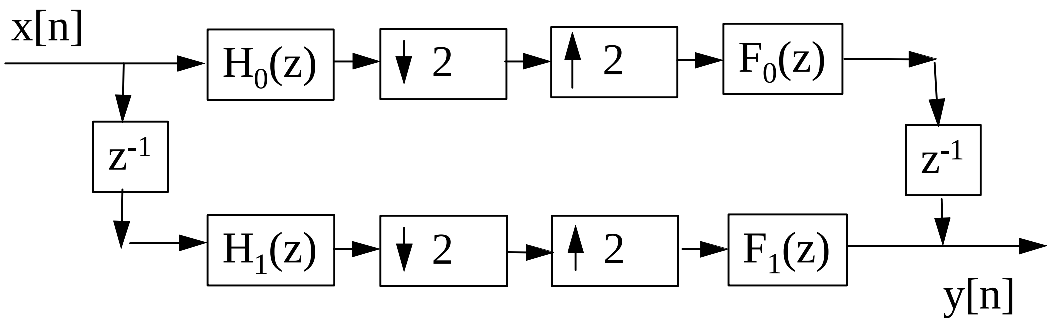
4. Polyphase Filters

Consider the modified QMF bank shown below. Assume that all the filters are FIR.



a) Express Y(z) in terms of X(z). Write Y(z) in the form: $Y(z) = T(z)X(z) + A(z)X(-z)$ (10 Points)

b) Show that using the substitutions $H_1(z) = H_0(-z)$ along with $F_0(z) = H_0(z)$ and $F_1(z) = H_1(z)$ will cancel out aliasing. Write the distortion transfer function T(z) in terms of $H_0(z)$. (7 Points)

c) Let $H_0(z)$ be a real coefficient linear phase FIR low pass filter of order N. Simplify T(z) and show that there is no phase distortion. Also show that N must be even to avoid $T\left(e^{j\pi/2}\right) = 0$ (8 points)

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5. FIR filters using the Window method

Consider the following ideal frequency response for a multiband filter:

$$H_d(w) = \begin{cases} 0 & 0 \le |w| < 0.3p \\ e^{-jwM/4} & 0.3p \le |w| < 0.6p \\ 0.5e^{-jwM/2} & 0.6p \le |w| \le p \end{cases}$$

a) Sketch the phase and magnitude response of this filter. (5 points)

b) Determine the ideal impulse response of this filter, $h_d[n]$ (7 points)

c) The impulse response $h_d[n]$ is multiplied by a Blackman window of length 50 resulting in a linear phase FIR system. Determine the set of approximation error specifications thare are satisfied by this FIR filter, (i.e. determine the parameters $d_1, d_2, d_3, A, B, w_{p1}, w_s, w_{p2}$).

$$\left|H\left(e^{jw}\right)\right| \le d_1 \qquad 0 \le w \le w_s$$

$$A - d_2 \le \left|H\left(e^{jw}\right)\right| \le A + d_2 \qquad w_{p1} \le w \le w_{p2}$$

$$B - d_3 \le \left|H\left(e^{jw}\right)\right| \le B + d_3 \qquad w_{p2} \le w \le p$$