

Final 13 December 1999  
EE524-Dr. Dickerson  
Answer all four problems

Name:

Social Security Number

Problem Number	Points	Score
1	30	
2	25	
3	25	
4	20	

Show all work for credit.

1. (Midterm 2)

Consider the comb filter

$$H(z) = \frac{1 - az^{-D}}{1 - a^4 z^{-4D}}$$

- a. Sketch the magnitude response and pole-zero diagram of this filter for  $|a| < 1$  and  $D=3$ .(13 points)
- b. Comment on the following conditions for this filter: all-pass, minimum phase, and linear phase. Which hold and don't hold and why? Answers without explanation will get zero points.(12 points)

2. Properties of DFT (Midterm 1)

Let  $x[n]$  be an  $N$ -point real sequence with the  $N$ -point DFT  $X[k]$  ( $N$  is even). In addition,  $x[n]$  satisfies the following symmetry property:

$$x\left(n + \frac{N}{2}\right) = -x(n) \quad n = 0, 1, \dots, \frac{N}{2} - 1$$

That is, the upper half of the sequence is the negative of the lower half.

a. Show that the even harmonics of the spectrum are zero: (10%)

$$X[k] = 0 \quad \text{for } k \text{ even}$$

b. Show that the values of this odd-harmonic spectrum can be computed by evaluating the  $N/2$  point DFT of a complex modulated version of the original sequence  $x[n]$  (10%)

$$X[2k+1] = \frac{N}{2} \text{ point DFT of } 2x[n]W_N^n$$

c. Prove the orthogonality property: (10%)

$$\sum_{n=0}^{N-1} W_N^{-kn} W_N^{ln} = \begin{cases} N & k = l \\ 0 & k \neq l \end{cases}$$

### 3. Exponentially Forgetting Window LMS Algorithm

Consider the time-varying cost function:

$$V(\mathbf{c}) = \sum_{k=0}^n \lambda^{n-k} e^2(k) \quad 0 < \lambda \leq 1$$

where  $\mathbf{c}$  is the tap-weight vector of a transversal FIR filter,  $e(n)$  is the estimation error defined by:

$$e(n) = y(n) - \mathbf{x}^T(n) \mathbf{c}(n-1)$$

a. Show that the weight update equation is

$$\mathbf{c}(n) = \mathbf{c}(n-1) + \mu (\mathbf{d}(n) - \mathbf{R}(n)\mathbf{c}(n-1))$$

$$\text{where: } \mathbf{d}(n) = \sum_{k=0}^n \lambda^{n-k} \mathbf{x}(k)y(k) \quad \text{and} \quad \mathbf{R}(n) = \sum_{k=0}^n \lambda^{n-k} \mathbf{x}(k)\mathbf{x}^T(n)$$

b. Using independence theory show that

4. Echo Cancellation

The LMS algorithm is used to implement an echo canceller with the block diagram shown below. Set up the equations that define the operation of this algorithm.