Midterm-November 6, 1998
EE524-Dr. Dickerson
Answer 4 out of 5 problems

Name:

Social Security Number

Problem Number	Points	Score
1	25	
2	25	
3	25	
4	25	
5	25	

Name:

1. Maximum Phase

A maximum phase system is obtained by reflecting all the zeros of the z-transform of a minimum phase system $H_{min}(z)$ to conjugate reciprocal positions outside the unit circle, i.e. replace each factor $\left(1-az^{-1}\right)$ with $\left(z^{-1}-a^{*}\right)$ where a denotes a single zero. Thus we can express the Z-transform of a maximum phase system as the product of $H_{min}(z)$ and an all-pass system $H_{ap}(z)$:

$$H_{\max}(z) = H_{\min}(z)\, H_{ap}(z)$$

For an (N-1) th order FIR filter with a real impulse response, we can write $H_{min}(z)$ as:

$$H_{\min}(z) = h(0)\prod_{k=1}^{N-1}\left(1-a_k z^{-1}\right) \qquad \left|a_k\right| < 1$$

a) Find an expression for $H_{ap}(z)$ (4 points)

b) Show that $H_{max}(z)$ can be expressed as

$$H_{\max}(z) = z^{-(N-1)} H_{\min}\left(z^{-1}\right) \qquad \text{(9 points)}$$

c) Use the result of part b to express the maximum phase sequence $h_{max}(n)$ in terms of the minimum phase sequence $h_{min}(n)$. (8 points)

d) Could either the maximum or minimum phase FIR filters be linear phase filters - why or why not? (4 points)

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2. Classical IIR Filter Design

a) Digitize an IIR lowpass filter using the bilinear transform with the transfer function given below using a sampling frequency of 1: (10 points)

$$H(s) = \frac{a}{s(s+a)} \qquad a = 0.5$$

b) Convert this to a bandpass filter with edges at 100 Hz and 200 Hz with a sampling frequency of 500 Hz. (10 points)

c) Can you use the impulse invariance method to design a highpass filter? Why or why not? Answer in terms of the mapping from the s-plane to the z-plane. (5 points)

Name:

3. DFTs

A signal consisting of a single sinusoid of frequency 2.5 Khz is sampled at 10kHz.

a) 128 samples of the signal are taken and the 128 point DFT is calculated. Give a definition of resolution in the DFT. What is the resolution in the 128 point DFT. Sketch the magnitude of the DFT. (10%)

b) Now zero pad the sequence to length 1024 points and take the 1024 point DFT. What is the resolution now? Sketch the DFT magnitude. (5%)

c) Take the result of step (b) (the zero-padded DFT $X_p(k)$) and form the sequence:

$$\bar{X}(k) = X_p(4k) \qquad k = 0,1,2,\ldots,255$$

Take the 256 point inverse DFT. Sketch the result. (10%)

Name:

4. FIR, IIR filters, system structure

An causal echo system repeats an attenuated version of the original signal after T seconds. The difference equation is:

$y(n) = x(n) + ax(n - M)$

a) If the signal is sampled at a rate of 100 KHz and the echo occurs after .01 seconds, what should be the value of M? (How many samples until the echo occurs?) (5 pts)

b) Write down the impulse response of the FIR system described in the difference equation. Give the system function and poles and zeros of this system for the M in part a? Draw the signal flow graph in direct form.(10 points)

c) A more realistic system would repeat the signal an infinite number of times reducing the amplitude by a factor of a at each repetition, with 0<a<1. The impulse response is:

$$h(n) = \sum_{k=0}^{\infty} a^k d(n - kM)$$

Find the system function and the poles and zeros of this system. Draw a signal flow diagram.(10 points)

Name:

5. FIR filters using the Window method

Consider the following ideal frequency response for a multiband filter:

$$H_d(w) = \begin{cases} e^{-jwM/2} & 0 \le |w| < 0.3p \\ 0 & 0.3p \le |w| < 0.6p \\ 0.5e^{-jwM/2} & 0.6p \le |w| \le p \end{cases}$$

a) Sketch the phase and magnitude response of this filter. (5 points)

b) Determine the ideal impulse response of this filter, $h_d[n]$ (7 points)

c) The impulse response $h_d[n]$ is multiplied by a Hamming window of length 75 resulting in a linear phase FIR system. Determine the set of approximation error specifications thare are satisfied by this FIR filter, (i.e. determine the parameters $d_1, d_2, d_3, A, B, w_{p1}, w_{s1}, w_{s2}, w_{p2}$).

$$A - d_1 \le \left|H\left(e^{jw}\right)\right| \le A + d_1 \qquad 0 \le w \le w_{p1}$$

$$\left|H\left(e^{jw}\right)\right| \le d_2 \qquad w_{s1} \le w \le w_{s2}$$

$$B - d_3 \le \left|H\left(e^{jw}\right)\right| \le B + d_3 \qquad w_{p2} \le w \le p$$