

Midterm-October 1, 1999
EE524-Dr. Dickerson
Answer all four problems

Name:

Social Security Number

Problem Number	Points	Score
1	30	
2	20	
3	25	
4	25	

Show all work for credit.

Name:

1. Properties of DFT

Let $x[n]$ be an N -point real sequence with the N -point DFT $X[k]$ (N is even). In addition, $x[n]$ satisfies the following symmetry property:

$$x\left[n + \frac{N}{2}\right] = -x[n] \quad n = 0, 1, \dots, \frac{N}{2} - 1$$

That is, the upper half of the sequence is the negative of the lower half.

a. Show that the even harmonics of the spectrum are zero: (10%)

$$X[k] = 0 \quad \text{for } k \text{ even}$$

b. Show that the values of this odd-harmonic spectrum can be computed by evaluating the $N/2$ point DFT of a complex modulated version of the original sequence $x[n]$ (10%)

$$X[2k+1] = \frac{N}{2} \text{ point DFT of } 2x[n]W_N^n$$

c. Prove the orthogonality property: (10%)

$$\sum_{n=0}^{N-1} W_N^{-kn} W_N^{ln} = \begin{cases} N & k = l \\ 0 & k \neq l \end{cases}$$

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2. Pole-zero filter design

Fetal monitors are used to monitor the heartbeat of a baby during labor. These filters must eliminate the heartbeat of the mother and strengthen the weaker heartbeat of the child. Use pole-zero placement to design a filter that will eliminate the heartbeat of the mother at 2 hertz and strengthen the heartbeat of the child at 3 hertz. The filter should be as flat as possible at other frequencies. The filter should have real coefficients. The sampling frequency is 24 Hertz

- a) Calculate the normalized digital frequencies of the maternal and fetal heartbeats. (4 %)
- b) Sketch the pole-zero diagram for this filter and write the transfer function of the filter in pole-zero form. Sketch the magnitude response of this filter. (16 points)

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3. DFTs

A signal, $x[n]$, consists of a single sinusoid of frequency $f = 0.5$ KHz is sampled at $f_s = 5$ kHz.

$$x[n] = \sin\left(\frac{2\pi f}{f_s} n\right)$$

- a) Define physical and computational resolution in a DFT. Clearly state how they are different. (5%)
- b) $N=32$ samples of $x[n]$ are taken and the 64 point DFT is calculated (the signal is zero-padded to length 64). What are the physical and computational resolution values in the DFT? Sketch the magnitude of the DFT. (10%) (Note: you do not need to work this out in detail, you should know the envelope from homework assignments and the classroom)
- c) Form the downsampled sequence ($N=32$):
 $y[n] = x[4n] \quad 0 \leq n \leq \lfloor (N-1)/4 \rfloor$
- Take the 64 point DFT. How is $Y[k]$ related to $X[k]$? What is the physical resolution of $Y[k]$? Sketch the magnitude response of $Y[k]$. (10%)

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4. FFT Algorithm

a. Describe how to compute a linear convolution for two finite length sequences using the FFT algorithm. Why would you use this algorithm instead of computing the linear convolution directly?(10 points).

b. Develop a flow-graph diagram for the basic radix-4 Decimation-in-Time algorithm for the $N=16$ case. The input is in digit reversed order and the output is in the normal order. Show the calculation for computing a N point DFT using 4 $N/4$ -point DFTs. (15 points)