

Following are sample questions from the Algebra, Trigonometry and Calculus examinations.

ALGEBRA

1. Simplify:

a) $\frac{\frac{1}{3} + \frac{1}{4}}{\frac{1}{6}}$ b) $3(3n+1) - (n+1)$ c) $\frac{(-2xy^2)^2}{4y^3}$ d) $\frac{1 + \frac{1}{y}}{\frac{1}{y} + \frac{1}{y^2}}$

e) $\frac{\sqrt{6} - \sqrt{12}}{\sqrt{3}} - \sqrt{2}$ f) $\frac{x^{\frac{1}{3}}y^{\frac{2}{3}}}{(2xy)^{\frac{1}{2}}}$ g) $\frac{a+ab}{a}$

h) $\frac{1}{x + \sqrt{x^2 + 1}} + \frac{1}{x - \sqrt{x^2 + 1}}$ i) $(3^{-2} + x^2)^{-1}$

2. Add, multiply, or divide, as indicated.

a) $(3x^2y)(7x^3y^5)$ b) $2a^2(5a-6)$ c) $\frac{x^3 - 6x^2 + 8x}{x-4}$
d) $(x+1)(x+2)(x+3)$ e) $x^2(2x^3 - 2x + 5) + (x^5 - x^4 + x^2 + 2)$

3. Factor:

a) $6x^2 + 13x + 6$ b) $a^2 - (b-a)^2$ c) $a^2 + ab + ac + bc$

4. Factor:

a) $10x^2 + 4x - 14$ b) $2x^3 + 3x^2 + x$
c) $(x+2)^2 3(x+3)^2 - (x+3)^3 2(x+2)$ d) $x^3 + 8$

5. Solve for x .

a) $6 = 3x - 5$ b) $\frac{x+2}{x+1} = 3$ c) $x(x+2) = 6(x+2)$

6. Solve for x and y in $\begin{cases} 5x + 4y = 2 \\ 7x - 3y = -23 \end{cases}$.

7. Factories A, B, and C produce 500, 800, and 1200 cars per week, respectively. What percentage of the cars would be produced by Factory A? What percentage of the cars would be produced by Factory A if Factory A were to double its production?

8. Three hamburgers and two soft drinks cost 293 cents while two hamburgers and three soft drinks cost 247 cents. Find the cost of one drink.

9. Find the equation of the line that contains the points $(-2, 3)$ and $(1, -2)$.

10. Sketch the points, x , on the x -axis satisfying $|2x - 3| < 7$.

11. Simplify:

a) $\frac{x^2 + 5x - 1}{2x + 1} - \frac{3}{2}x$

b) $1 + x[2 - x(3 - x(5 + x))]$

c) $\sqrt{xy} \left(\sqrt{\frac{x}{y}} - \sqrt{\frac{y}{x}} \right)$

d) $\left(25 \frac{c^{-6}}{d^{-4}} \right)^{\frac{3}{2}}$

e) $\frac{(a+b)^2 - a^2 - b^2}{a+b}$

f) $10^{\log_{10} x}$

g) $F(3) - F(1)$ where $F(t) = \frac{2}{3}(t-1)^{\frac{3}{2}} + c$

h) $\frac{(x^2 + 1)\frac{1}{2}x^{\frac{1}{2}} - x^{\frac{1}{2}}2x}{1 - 3x^2}$

i) $\frac{\sqrt[3]{16x^2y}}{\sqrt[3]{6x}}$

12. Add, multiply, or divide:

a) $\frac{1}{(x+1)(x+2)} + \frac{-1}{(x+2)(x+3)}$

b) $(b^2 + b)(b^{-2} + b^{-1})$

c) $(x^3 + 2) \div (x^2 + 3x + 2)$

13. Solve for x :

a) $\frac{5(x-2) - 3(x-4)}{x-2} = 2$

b) $(x+1)^2 - x^2 = 0$

c) $x(x-1) \leq 6$

d) $\sqrt{x+2} - \sqrt{x-3} = 1$

e) $2x^3 + 5x^2 - 4x - 3 = 0$

f) $(1+x)^4 = 2$

g) $\frac{x^2 + 1}{x^2 - 1} = y$

h) $\log_y x + \log_y (x+3) = 1$

14. Find (x, y) satisfying:

a) $\frac{2}{x} + \frac{2}{y} = 6$
 $\frac{3}{x} + \frac{2}{y} = 3$

b) $xy = 12$
 $x + 3y = 13$

15. Find numbers a , b , and c such that the graph of the parabola $y = ax^2 + bx + c$ passes through $(-1, 1)$, $(0, 1)$, and $(1, 2)$.

16. Sketch well labeled graphs of:

a) $y = -\frac{1}{2}x + 1$

b) $y = \sqrt{x}$

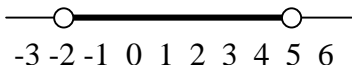
c) $y = \frac{x+1}{x-1}$

17. Find the point of the line $y = 2x - 5$ that is nearest the origin.

18. The height of a garage door is 6.8 feet greater than the width. The diagonal of the door is 10 feet. What are the dimensions of the door?

The following answers are for the sample Algebra Placement Exam.

ANSWERS (Algebra)

1. a) $\frac{7}{2}$
 b) $8n+2$
 c) x^2y
 d) y
 e) -2
 f) $\frac{y^{1/6}}{2^{1/2}x^{1/6}}$ or $\frac{2^{1/2}x^{5/6}y^{1/6}}{2x}$ or $2^{-1/2}x^{-1/6}y^{1/6}$
 g) $1+b$
 h) $-2x$
 i) $\frac{9}{1+9x^2}$
2. a) $21x^5y^6$
 b) $10a^3-12a^2$
 c) x^2-2x
 d) $x^3+6x^2+11x+6$
 e) $3x^5-x^4-2x^3+6x^2+2$
3. a) $(3x+2)(2x+3)$
 b) $b(2a-b)$
 c) $(a+c)(a+b)$
4. a) $2(5x+7)(x-1)$
 b) $x(2x+1)(x+1)$
 c) $x(x+2)(x+3)^2$
 d) $(x+2)(x^2-2x+4)$
5. a) $x=\frac{11}{3}$
 b) $x=-\frac{1}{2}$
 c) $x=6$ or $x=-2$
6. $x=-2, y=3$
7. 20% by Factory A currently, $33\frac{1}{3}\%$ if production doubled.
8. \$0.31
9. $\frac{y-3}{x+2}=-\frac{5}{3}$ or $y-3=-\frac{5}{3}(x+2)$ or $y=-\frac{5}{3}x-\frac{1}{3}$ or $5x+3y+1=0$
10. $-2 < x < 5$

11. a) $\frac{-4x^2+7x-2}{2(2x+1)}$
 b) $1+2x-3x^2+5x^3+x^4$
 c) $x-y$
 d) $125\frac{d^6}{c^9}$
 e) $\frac{2ab}{a+b}$
 f) x
 g) $\frac{4\sqrt{2}}{3}$

h) $\frac{1}{2x^{1/2}}$

i) $2\sqrt[3]{\frac{xy}{3}}$ or $\frac{2}{3}\sqrt[3]{9xy}$

12. a) $\frac{2}{(x+1)(x+2)(x+3)}$

b) $b+2+b^{-1}$ or $b+2+\frac{1}{b}$ or $\frac{b^2+2b+1}{b}$
or $\frac{(b+1)^2}{b}$

c) $\frac{x^3+2}{x^2+3x+2}$ or $x-3+\frac{7x+8}{x^2+3x+2}$

13. a) No solution

b) $x = -\frac{1}{2}$

c) $-2 \leq x \leq 3$

d) $x = 7$

e) $x = 1$, $x = -\frac{1}{2}$, or $x = -3$

f) $x = -1 \pm \sqrt[4]{2}$ or $x = -1 \pm \sqrt[4]{2}i$

g) $x = \pm \sqrt{\frac{y+1}{y-1}}$

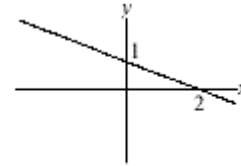
h) $x = \frac{-3 \pm \sqrt{9+4y}}{2}$

14. a) $x = -\frac{1}{3}$, $y = \frac{1}{6}$

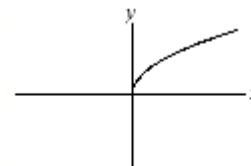
b) $x = 4$, $y = 3$ or $x = 9$, $y = \frac{4}{3}$

15. $a = \frac{1}{2}$, $b = \frac{1}{2}$ and $c = 1$

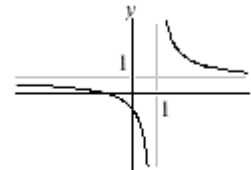
16. a) $y = -\frac{1}{2}x + 1$



b) $y = \sqrt{x}$



c) $y = \frac{x+1}{x-1}$



17. $(2, -1)$

18. 2.8 feet by 9.6 feet

TRIGONOMETRY

1. In separate, standard (x, y) -coordinate systems, with all angles in standard position, sketch the terminal sides of the angles.

a) 165° b) $\frac{4\pi}{3}$ (radians) c) -75° .

2. Find the length of the arc cut from a circle of radius 10 cm by the sides of a central angle (vertex at the center of the circle) of

a) 25° b) 0.5 (radians).

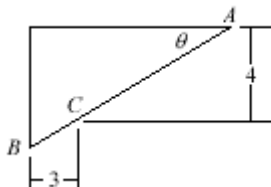
3. a) Consider a right triangle with angles A , B and C and opposite sides a , b , c , respectively. If C is 90° , find

i) $\sin A$ ii) $\tan B$ iii) $\csc B$
(express answers in terms of a , b , and c).

- b) If $c = 5$ and $b = 1$, find

i) $\cos A$ ii) $\cot A$ iii) $\sec B$
(Do not use a calculator – use radicals or fractions.)

4. Compute the distance AB shown in the figure, given that $\theta = 30^\circ$.



5. a) In a standard (x, y) -coordinate system a unit circle is drawn, with center at the origin. An angle θ is sketched, in standard position, and its terminal side meets the circle in the point (p, q) . What are the sine, cosine, and tangent of θ , in terms of p and q ?

- b) Compute, without calculator,

i) $\sin 300^\circ$ ii) $\sec\left(\frac{13\pi}{6}\right)$.

6. In triangle ABC , $A = 65^\circ$, $b = 9$, and $a = 10$. Find B .

7. The diagonals of a parallelogram are 20 cm and 30 cm long and intersect at an angle of 25° . Find the sides of the parallelogram.

8. Complete the following standard identities:

a) $\sin 2\theta =$ b) $\cos^2 \theta =$ (trig functions of 2θ) c) $\tan^2 \theta + 1 =$

9. Verify that the following are identities:

a) $\tan\left(\frac{x}{2}\right) = (\sec x - 1)/(\sin x \sec x)$ b) $\cos 3x = 4\cos^3 x - 3\cos x$

10. Find all solutions x in the interval $[0, 2\pi]$ for each of the following equations:

a) $\tan x = -\frac{1}{2}$ b) $2\sin x \cos^2 x - \sin^3 x = 0$.

11. Graph, separately, one period of

a) $y = \sin 2x$ b) $y = 0.5 \cos\left(\pi x - \frac{\pi}{4}\right)$

12. a) Sketch $y = \cos^{-1} x$.

b) Compute, without a calculator:

i) $\sin^{-1}\left(-\frac{1}{2}\right)$ ii) $\sin\left(2\sin^{-1}\left(\frac{3}{4}\right)\right)$

c) Solve for all x 's in $[0, 2\pi]$ satisfying $\sin(\cos^{-1} x) = x$.

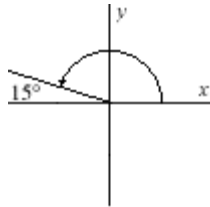
13. If $x = 2 \tan u$, write $\frac{u}{16} + (\sin 2u)/32$ in terms of x .

14. Find, without a calculator, the real part of $(\cos 30^\circ + i \sin 30^\circ)^{10}$.

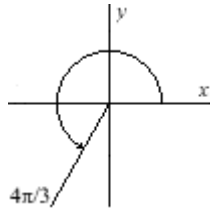
The following answers are for the sample Trigonometry Placement Exam.

ANSWERS (Trigonometry)

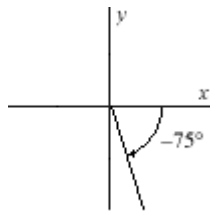
1. a)



b)



c)



2. a) $\frac{25\pi}{18}$ cm.

b) 5 cm.

3. a) i) $\sin A = \frac{a}{c}$

ii) $\tan B = \frac{b}{a}$

iii) $\csc B = \frac{c}{b}$

b) i) $\cos A = \frac{1}{5}$

ii) $\cot A = \frac{\sqrt{6}}{12}$

iii) $\sec B = \frac{5\sqrt{6}}{12}$

4. $AB = 8 + 2\sqrt{3}$

5. a) $\sin \theta = q$, $\cos \theta = p$, $\tan \theta = \frac{q}{p}$

b) i) $\sin 300^\circ = -\frac{\sqrt{3}}{2}$

ii) $\sec \frac{13\pi}{6} = \frac{2\sqrt{3}}{3}$

6. $\sin B = (9 \sin 65^\circ)/10$

7. $a^2 = 325 - 300 \cos(25^\circ)$

$a^2 = 325 - 300 \cos(155^\circ)$

8. a) $\sin 2\theta = 2 \sin \theta \cos \theta$

b) $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$

c) $\tan^2 \theta + 1 = \sec^2 \theta$

9. a)

$$\frac{\sec x - 1}{\sin x \sec x} = \frac{1 - \cos x}{\sin x} = \frac{2 \sin^2 \frac{1}{2} x}{2 \sin \frac{1}{2} x \cos \frac{1}{2} x} = \tan \frac{1}{2} x$$

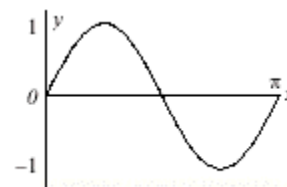
b)

$$\begin{aligned} \cos 3x &= \cos(x + 2x) = \cos x \cos 2x - \sin x \sin 2x \\ &= \cos x(2 \cos^2 x - 1) - 2 \cos x(1 - \cos^2 x) \\ &= 4 \cos^3 x - 3 \cos x \end{aligned}$$

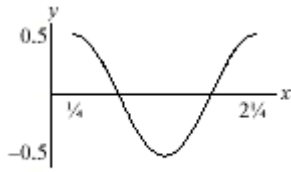
10. a) $x = 5.82$ or 2.68

b) $\sin^2 x = 2/3$

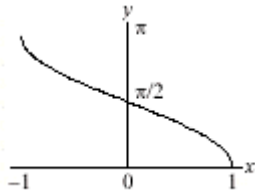
11. a)



b)



12. a)



b) $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$,

$$\sin\left[2\sin^{-1}\left(\frac{3}{4}\right)\right] = \frac{3\sqrt{7}}{8}$$

c) $x = \frac{\sqrt{2}}{2}$

13. $\frac{1}{16}\tan^{-1}\frac{x}{2} + \frac{1}{8}\frac{x}{x^2+4}$

14. $\frac{1}{2}$

The following questions are similar to the questions in the Calculus Placement Exam.

CALCULUS

1. If $f(x) = 1 - 3 \cos x$, find $f'\left(\frac{\pi}{2}\right)$.

2. If $f(x) = \sin(\pi x)$, find $f'\left(\frac{1}{2}\right)$.

3. Find the first derivative of the following functions:

a) $f(x) = \sqrt{1 + \sin x}$ b) $f(x) = \tan\left(\frac{\pi}{x^2}\right)$ c) $f(x) = \frac{\sin x}{x^2 + 2}$ d) $f(x) = x \cos 2x$.

4. Find the equation of the line tangent to the graph of $f(x) = \sqrt{x+1}$ at the point (3,2).

5. Compute the indicated limits

a) $\lim_{x \rightarrow 3} \frac{x^2 - 6x + 9}{x^2 - 2x - 3}$ b) $\lim_{x \rightarrow 1^-} \frac{|x-1|}{x-1}$ c) $\lim_{x \rightarrow 0} \frac{x}{\sin(\pi x)}$.

6. Suppose $h(x) = \frac{x}{x^2 + 1}$.

- For which intervals h is increasing?
- For what x will h have a relative minimum?
- Find the horizontal asymptote for the graph of h .
- Sketch the graph of h .

7. If $f(x) = \frac{1}{\sec x}$ for $x \neq \frac{\pi}{2}$, find a value for $f\left(\frac{\pi}{2}\right)$ which will make f continuous at $\frac{\pi}{2}$.

8. If $-3 \sin x = \tan y + y$, find $\frac{dy}{dx}$ at the point (0, 0).

9. A child flies a kite at a height of 300 feet. The wind carries the kite horizontally at a rate of 10 ft/sec. How fast must the child play out string when the kite is 500 feet from him?

10. Two positive numbers have the property that their product is 2 and their sum is as small as possible. Find their sum.

The following answers are for the sample Calculus Placement Exam.

ANSWERS (Calculus)

$$1. \quad f(x) = 1 - 3 \cos x$$

$$f'(x) = -3(-\sin x) = 3 \sin x$$

$$f'\left(\frac{\pi}{2}\right) = 3 \cdot 1 = 3$$

$$2. \quad f(x) = \sin(\pi x)$$

$$f'(x) = \pi \cos(\pi x)$$

$$f''(x) = -\pi^2 \sin(\pi x)$$

$$f''\left(\frac{1}{2}\right) = -\pi^2 \sin \frac{\pi}{2} = -\pi^2$$

$$3. \quad a) \quad f(x) = \sqrt{1 + \sin x}$$

$$f'(x) = \frac{\cos x}{2\sqrt{1 + \sin x}}$$

$$b) \quad f(x) = \tan\left(\frac{\pi}{x^2}\right)$$

$$f'(x) = \sec^2\left(\frac{\pi}{x^2}\right) \cdot \frac{d}{dx}\left(\frac{\pi}{x^2}\right)$$

$$= \frac{-2\pi}{x^3} \sec^2\left(\frac{\pi}{x^2}\right)$$

$$c) \quad f(x) = \frac{\sin x}{x^2 + 2}$$

$$f'(x) = \frac{(x^2 + 2)\cos x - 2x \sin x}{(x^2 + 2)^2}$$

$$d) \quad f(x) = x \cos 2x$$

$$f'(x) = x \cdot \frac{d}{dx} \cos 2x + (\cos 2x) \cdot \frac{dx}{dx}$$

$$= -2x \sin 2x + \cos 2x$$

$$4. \quad f(x) = \sqrt{x+1}$$

$$f'(x) = \frac{1}{2\sqrt{x+1}}$$

$$f'(3) = \frac{1}{2\sqrt{3+1}} = \frac{1}{4}$$

$$\therefore \text{tangent line is } y - 2 = \frac{1}{4}(x - 3)$$

$$5. \quad a) \quad \lim_{x \rightarrow 3} \frac{x^2 - 6x + 9}{x^2 - 2x - 3}$$

$$= \lim_{x \rightarrow 3} \frac{(x-3)^2}{(x-3)(x+1)}$$

$$= \lim_{x \rightarrow 3} \frac{x-3}{x+1}$$

$$= 0$$

$$b) \quad \lim_{x \rightarrow 1^-} \frac{|x-1|}{x-1}$$

$$= \lim_{x \rightarrow 1^-} \frac{1-x}{x-1}$$

$$= \lim_{x \rightarrow 1^-} \frac{-(x-1)}{x-1}$$

$$= -1$$

$$c) \quad \lim_{x \rightarrow 0} \frac{x}{\sin(\pi x)}$$

$$= \lim_{x \rightarrow 0} \frac{\pi x}{\sin(\pi x)} \cdot \frac{1}{\pi}$$

$$= 1 \cdot \frac{1}{\pi} = \frac{1}{\pi}$$

$$6. \quad a) \quad h(x) = \frac{x}{x^2 + 1}$$

$$h'(x) = \frac{(x^2 + 1)(1) - x(2x)}{(x^2 + 1)^2}$$

$$= \frac{-x^2 + 1}{(x^2 + 1)^2}$$

$h'(x) > 0$ when $-x^2 + 1 > 0$ or $x^2 < 1$;
therefore $-1 < x < 1$

b) $h'(x) = 0$ when $-x^2 + 1 = 0$ or $x = \pm 1$. At $x = +1$ there is a relative maximum because $h'(x) > 0$ to the left of 1 and $h'(x) < 0$ to the right of 1. At $x = -1$, there is a relative minimum because $h'(x) < 0$ to the left of -1, but $h'(x) > 0$ right of -1.

c) $\lim_{x \rightarrow \infty} h(x)$

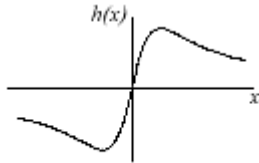
$$= \lim_{x \rightarrow \infty} \frac{x}{x^2 + 1}$$

$$= \lim_{x \rightarrow \infty} \frac{1}{x + \frac{1}{x}}$$

$$= 0$$

Hence, $y = 0$ is a horizontal asymptote.

d)



7. $\lim_{x \rightarrow \frac{\pi}{2}} f(x) = \lim_{x \rightarrow \frac{\pi}{2}} \frac{1}{\sec x} = \lim_{x \rightarrow \frac{\pi}{2}} \cos x = 0$

Use $f\left(\frac{\pi}{2}\right) = 0$.

8. $-3 \sin x = \tan y + y$

Differentiate implicitly:

$$-3 \cos x = (\sec^2 y) \frac{dy}{dx} + \frac{dy}{dx}$$

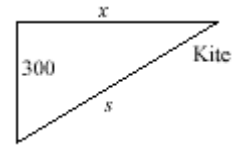
When $x = 0$, $y = 0$, so

$$-3 \cos 0 = (\sec^2 0) \frac{dy}{dx} + \frac{dy}{dx}$$

$$-3 = 2 \frac{dy}{dx}$$

$$\left. \frac{dy}{dx} \right|_{(0,0)} = -\frac{3}{2}$$

9. $s^2 = 300^2 + x^2$.



Differentiate with respect to time t :

$$2s \frac{ds}{dt} = 2x \frac{dx}{dt} \text{ . We want } \frac{ds}{dt} \text{ when } s = 500$$

$$\text{and } \frac{dx}{dt} = 10 \text{ . So, } \frac{ds}{dt} = \frac{x}{s} \frac{dx}{dt} = \frac{\sqrt{s^2 - 300^2}}{s} \frac{dx}{dt}$$

$$= \frac{\sqrt{500^2 - 300^2}}{500} (10) = 8 \text{ ft./sec.}$$

10. If x and y are the numbers, $xy = 2$. So, $y = \frac{2}{x}$.

We want to minimize $x + y = x + \frac{2}{x}$. At a

critical point, $\frac{d}{dx} \left(x + \frac{2}{x} \right) = 1 - \frac{2}{x^2} = 0$. So,

$x^2 = 2$, $x = \pm\sqrt{2}$. Since $x > 0$, $x = +\sqrt{2}$ and

$y = \frac{2}{\sqrt{2}} = \sqrt{2}$. Their sum is $2\sqrt{2}$.